Towards a Formalization of Responsibility

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3rd International Workshop on Normative Multiagent Systems
Luxembourg, 15 July 2008

Responsibility

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The meaning of the term 'responsibility' in this example implies the duty, or the obligation, to ensure that each account balance will be positive.

This is compatible with the following definition, suggested by [Santos and Carmo, 1996].

Definition (Notion 1). Agent a is responsible for φ if and only if a is obliged to ensure that φ.

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Example (continuation). On Tuesday the balance of account 1 is 10,000 Euro, while the balance of account 2 is only 50 Euro! Moreover, the company will spend 5,000 Euro from account 2 either on Tuesday or Wednesday.

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So, Alice must make a decision. In particular, she has the choice between making a transfer from account 1 to account 2 on Tuesday or wait until Wednesday.

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Example (continuation). Alice decides leave the transfer to Wednesday. However, the company spends the money on Tuesday, and therefore she hears from the director: "— You are *responsible* for the balance of account 2 is negative!"

The meaning of the term 'responsibility' in this case implies blameworthiness, or the guilty of the negative balance.

The latter is compatible with the following definition, based on [Kein, 1993] and [Heinaman, 1993].

Definition (Notion 2). Agent a is responsible for φ if and only if a freely, knowingly and intentionally behaves in such a way that is necessary for the occurrence of a "wrong" consequence φ.

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In this work we try to build a framework wherein one can formalize these two notions and capture the relation between them.

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- $V : Atm \to 2^W$ is the interpretation of atoms.

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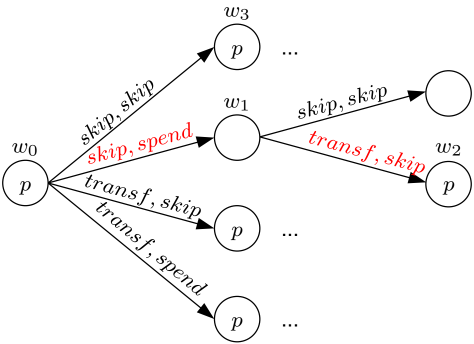

Σ contains: $\sigma = \{(w_0, \alpha_0), (w_1, \alpha_1), \ldots\}$.

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- if $\varphi \in \mathcal{L}_s$ then $\neg\varphi \in \mathcal{L}_s$;
- if $\varphi_1, \varphi_2 \in \mathcal{L}_s$ then $\varphi_1 \vee \varphi_2 \in \mathcal{L}_s$;
- if $\sigma \in \Sigma$, $C \subseteq Agt$ and $\psi \in \mathcal{L}_p$ then $[C{:}\sigma]\psi \in \mathcal{L}_s$;
- if $C \subseteq Agt$ and $\psi \in \mathcal{L}_p$ then $\langle\!\langle C \rangle\!\rangle\psi \in \mathcal{L}_s$;

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and $\mathcal{L}_p$ is defined by:

- if $\varphi \in \mathcal{L}_s$ then $\mathrm{X}\varphi, \mathrm{G}\varphi \in \mathcal{L}_p$;
- if $\varphi_1, \varphi_2 \in \mathcal{L}_s$ then $\varphi_1 \mathrm{U} \varphi_2 \in \mathcal{L}_p$.

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These are not well-formed formulas:

$\mathrm{GX}p$

$[C{:}\sigma][C{:}\sigma]p$

$\langle\!\langle C \rangle\!\rangle[C{:}\sigma]p$

$[C{:}\sigma]\langle\!\langle C \rangle\!\rangle p$

Language

Intended meanings:

$\mathrm{X}\varphi$: 'φ is true in the next state'.

$\mathrm{G}\varphi$: 'φ is true from the current state on'.

$\varphi_1 \mathrm{U} \varphi_2$: '$\varphi_1$ is true from the current state on until φ_2 is true'.

$\langle\langle C \rangle\rangle \psi$: 'coalition C has the power of bringing about ψ'.

$[C{:}\sigma]\psi$: 'if coalition C follows strategy σ then ψ is true'.

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$\Lambda(w, C{:}\sigma)$ denotes the set of all computations such that for each $a \in C$ and each pair (w_i, α_i) in the sequence we have $(\sigma(w_i))(\alpha_i) = \alpha_i(a)$
(i.e., it denotes the set of all computations starting at w such that coalition C follows strategy σ).

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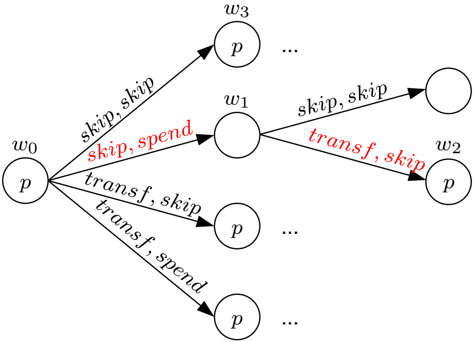

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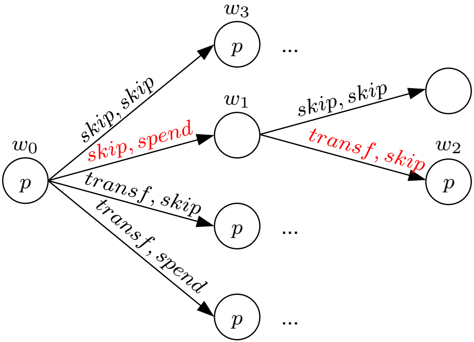

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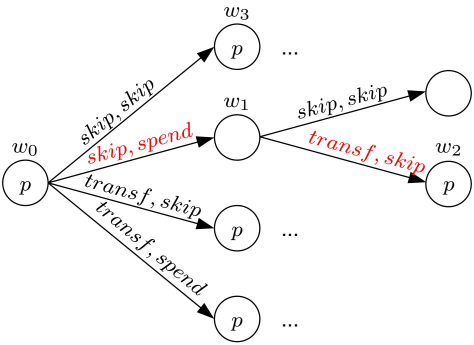

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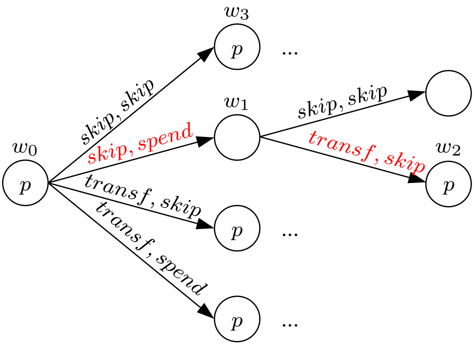

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Note that $\Lambda(w, \emptyset{:}\sigma) = \Lambda(w)$ and $\Lambda(w, Agt{:}\sigma)$ is a singleton.

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$\mathcal{M}, w \models [C{:}\sigma]\psi$	iff	for all $\lambda \in \Lambda(w, C{:}\sigma)$ we have $\mathcal{M}, \lambda \models \psi$
$\mathcal{M}, w \models \langle\langle C \rangle\rangle\psi$	iff	there is $\sigma \in \Sigma$ such that
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$\mathcal{M}, \lambda \models \mathrm{X}\varphi$	iff	$\mathcal{M}, w_1 \models \varphi$
$\mathcal{M}, \lambda \models \mathrm{G}\varphi$	iff	for all $i \in \mathbb{N}$ we have $\mathcal{M}, w_i \models \varphi$
$\mathcal{M}, \lambda \models \varphi_1 \mathrm{U} \varphi_2$	iff	there is $i \in \mathbb{N}$ such that $\mathcal{M}, w_i \models \varphi_2$ and
		for all $k \in \mathbb{N}$ if $0 \leq k < i$ then $\mathcal{M}, w_k \models \varphi_1$

CATL

CATL model checking is in PTIME [van der Hoek et al., 2005].

CATL satisfiability checking is in EXPTIME [Walther et al., 2007].

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Second, models are as before, but:

- $V : Atm \cup Atm_v \to 2^W$,
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(The latter is equivalent to the axiom scheme $\neg\langle\langle\emptyset\rangle\rangle \mathrm{X} v_C$.)

Then obligations can be defined as abbreviations:

$$\mathrm{OX}_C\varphi \overset{\text{def}}{=} \langle\langle\emptyset\rangle\rangle\mathrm{X}(\neg\varphi \to v_C)$$

$$\mathrm{OG}_C\varphi \overset{\text{def}}{=} \langle\langle\emptyset\rangle\rangle\mathrm{G}(\neg\varphi \to v_C)$$

where φ is a state formula.

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In our framework it can be defined by:

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Note that $\mathrm{A}_C\psi$ must be a path formula.

For example, if we want to check whether coalition C attempts to bring about $\mathrm{X}\varphi$, it is necessary to look at the joint action that C will execute in the current state. Therefore, $\mathrm{A}_C\mathrm{X}\varphi$ cannot be evaluated in a single state of the system. Rather, it should be evaluated in a “run of the system”. In CATL terms: it should be evaluated in a computation.

Adding Intentional Behaviour

Formally:

Let $\lambda = w_0, \alpha_0, w_1, \alpha_1, \ldots$, and
let $\sigma = \{(w_0, \alpha_0), (w_1, \alpha_1), \ldots\}$.

$\mathcal{M}, \lambda \models \mathrm{A}_C \psi$ if and only if

for all $\lambda' \in \Lambda(w_0, C{:}\sigma)$ we have $\mathcal{M}, \lambda' \models \psi$ and
there is $\sigma' \in \Sigma$ and $\lambda'' \in \Lambda(w_0, C{:}\sigma')$ such that $\mathcal{M}, \lambda'' \not\models \psi$

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there is $\lambda' \in \Lambda(w_0, C{:}\sigma)$ such that $\mathcal{M}, \lambda' \models \psi$ and
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We express that
'If C follows σ then C attempts to bring about φ in the next state', by:

$$\mathrm{AX}_{C:\sigma}\varphi \stackrel{\text{def}}{=} ([C{:}\sigma]\mathrm{X}\varphi \wedge \langle\!\langle Agt \rangle\!\rangle \mathrm{X}\neg\varphi) \vee (\neg[C{:}\sigma]\mathrm{X}\neg\varphi \wedge \langle\!\langle C \rangle\!\rangle \mathrm{X}\neg\varphi)$$

and we express that
'If C follows σ then C attempts to bring about φ from now on', by:

$$\mathrm{AG}_{C:\sigma}\varphi \stackrel{\text{def}}{=} ([C{:}\sigma]\mathrm{G}\varphi \wedge \langle\!\langle Agt \rangle\!\rangle (\top \mathrm{U} \neg\varphi)) \vee (\neg[C{:}\sigma](\top \mathrm{U} \neg\varphi) \wedge \langle\!\langle C \rangle\!\rangle (\top \mathrm{U} \neg\varphi))$$

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$\mathrm{BRX}_{C:\sigma}v_C$ is read: ‘if C follows σ then C is backward-looking responsible for v_C in the next state’.

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$\mathrm{BRX}_{C:\sigma} v_C$ is read: 'if C follows σ then C is backward-looking responsible for v_C in the next state'.

We define it only for violations because of the "wrong-doing" condition.

Forward-Looking vs. Backward-Looking

The following formula is valid:

$$(\mathrm{FRX}_C\varphi \land [C{:}\sigma]\mathrm{X}\neg\varphi) \rightarrow \mathrm{BRX}_{C:\sigma}v_C)$$

If C is held forward-looking responsible for φ and C follows a strategy that leads to a failure then C is backward-looking responsible for it.

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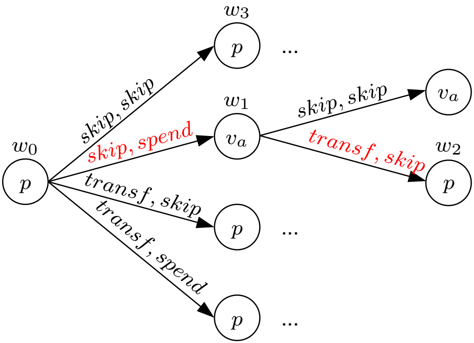
Proof. Indeed since:
$\mathcal{M}, w \models \mathrm{OX}_C\varphi$ iff $\mathcal{M}, w \models \langle\langle\emptyset\rangle\rangle\mathrm{X}(\neg\varphi \rightarrow v_C)$.
Then $\mathcal{M}, w \models \mathrm{OX}_C\varphi \wedge [C{:}\sigma]\mathrm{X}\neg\varphi$ implies $\mathcal{M}, w \models [C{:}\sigma]\mathrm{X}v_C$.
Moreover, remember that $\mathcal{M}, w \models \neg\langle\langle\emptyset\rangle\rangle\mathrm{X}v_C$,
which implies $\mathcal{M}, w \models \langle\langle Agt\rangle\rangle\mathrm{X}\neg v_C$.
Therefore, $\mathcal{M}, w \models [C{:}\sigma]\mathrm{X}v_C \wedge \langle\langle Agt\rangle\rangle\mathrm{X}\neg v_C$,
which immediately implies $\mathcal{M}, w \models \mathrm{BRX}_{C:\sigma}v_C$. □

Forward-Looking vs. Backward-Looking

For example,



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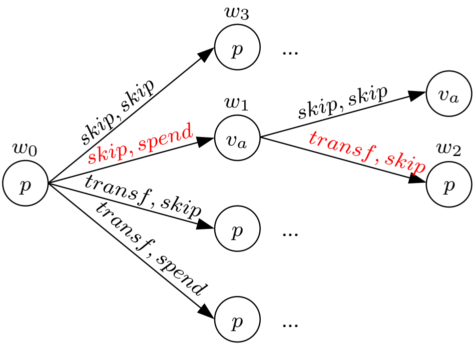

We have $\mathcal{M}, w_0 \models \langle\langle \emptyset \rangle\rangle \mathrm{X}(\neg p \to v_a) \wedge \langle\langle a \rangle\rangle \mathrm{X} p$.

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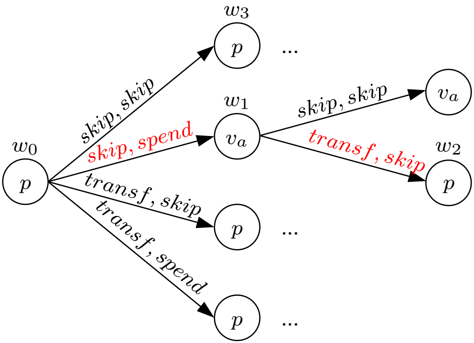

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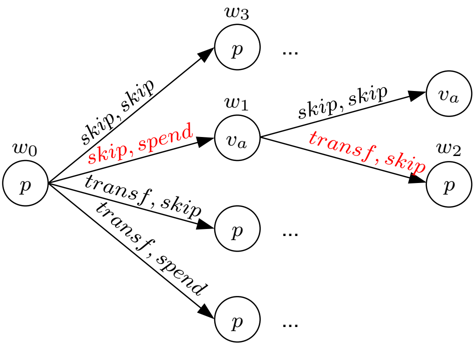

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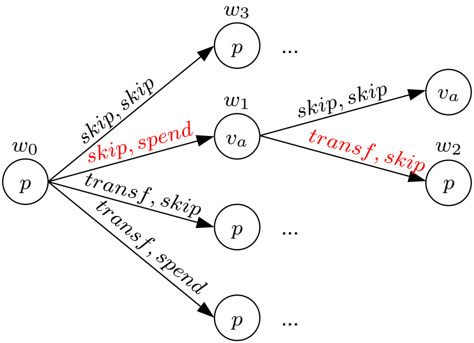

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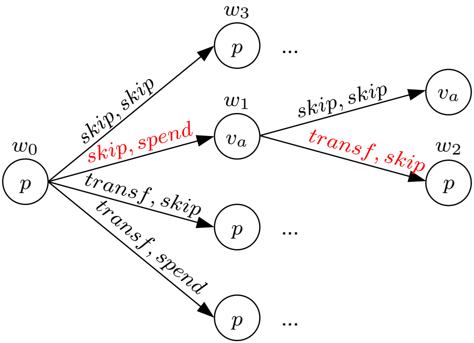

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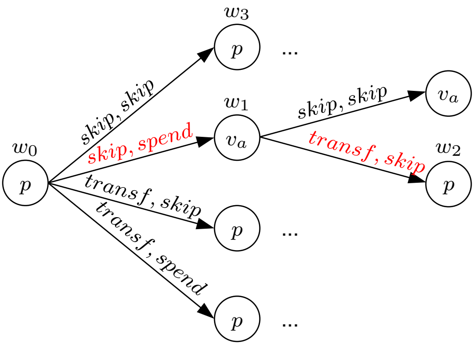

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Then we have $\mathcal{M}, w_0 \models [a{:}\sigma] v_a \wedge \mathrm{AX}_{a:\sigma} v_a$.
Therefore, $\mathcal{M}, w_0 \models \mathrm{BRX}_{a:\sigma} v_a$.

Future Works

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Forward-looking responsibility inherits from obligations the distinction between ought-to-do and ought-to-be statements.

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We can also define dynamic obligations as abbreviations:

$$\mathrm{OX}_C(\sigma) \stackrel{\text{def}}{=} [C{:}\overline{\sigma}]\mathrm{X}v_C$$

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We need to define operations over strategies, similar to PDL.

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The addition of an operator K for knowledge in ATL was already proposed by [van der Hoek and Wooldridge, 2003].

However in our framework there are some technical problems to be solved. For instance its interaction with obligations.

Thank you!